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# OPTIMAL LEVERAGE: A DYNAMIC ALLOCATION APPROACH TO CAPTURING THE OPTIMAL RISK-RETURN RATIO



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## INTRODUCTION

Investing in a risky asset is, as the appellation suggests, risky. It is this risk that drives the expected return and that entices investors into investing in this risky asset. It is intuitive to think that the higher the risk, the higher the expected return. However, research shows that risky assets do not perform as well as the less risky ones on the long run (Baker et al. 2010). Wild fluctuations in an asset value require a significant upside bias to remain even in value. This is due to the lack of symmetry between a percentage gain and loss.

## MARKET PERFORMANCE

It is quite clear from market performances over the last decade that a buy-and-hold allocation is not very efficient, even on a diversified asset such as a market-capitalization weighted index. As we can see in the chart below, a cash investment would have had a much smaller volatility compared to the EURO STOXX 50 Index and would have had better returns.

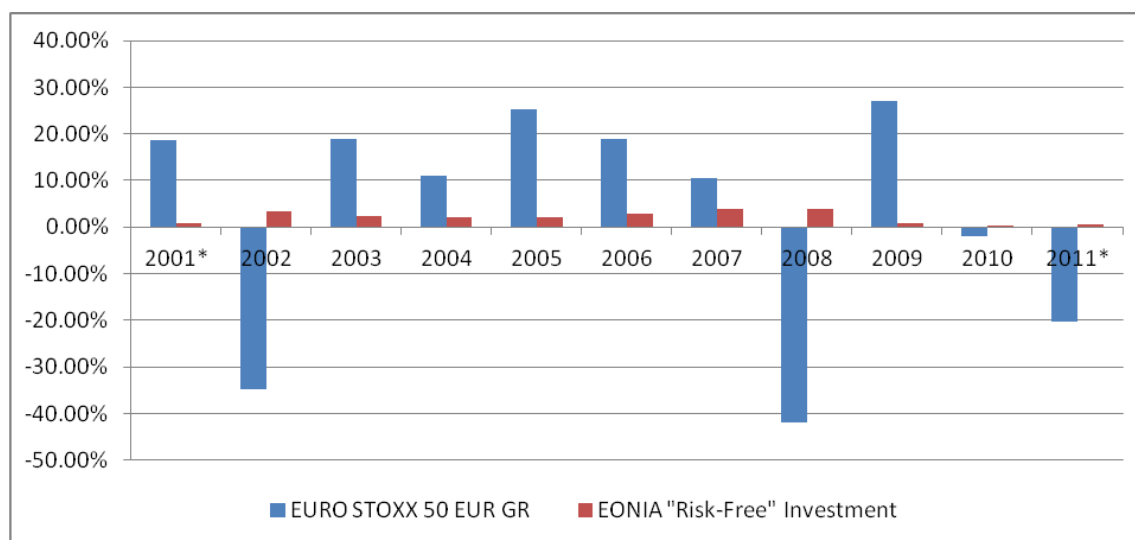


Figure 1: Returns of the EURO STOXX 50 vs. Cash. \*Incomplete year

However, we can also see that there are time periods when the risky asset outperforms the cash investment, therefore a cash investment could have been outperformed using a selective investment in the EURO STOXX 50.

## INVESTMENT POSSIBILITIES

Once an investor chooses a risky asset they would like to have exposure on, they must choose their allocation to it. There are three possibilities, zero investment, full investment, a proportion of the portfolio into the risky asset.

A zero investment allocation is very interesting from the risk perspective, as it would generate "risk-free" interest from holding cash. However, the excess return and hence the risk-return ratio are zero.

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A full investment allocation will contain the full risk of the risky asset. The risk-return ratio of the portfolio will be completely at the mercy of the underlying asset.

A partial investment in the risky asset would yield returns as an average of the risky asset and a cash position. It would also have reduced volatility compared to a full risky investment. However, if the proportion invested is constant through time, the resulting portfolio's performance is a simple average of the two underlyings and will consistently be subpar to one of the other two investment possibilities.

We therefore look to adapt allocation dynamically to improve the risk-return ratio of the portfolio. This will intuitively have more of the portfolio invested in the risky asset at times when the asset has better risk-weighted prospects, and hold more cash in the opposite case. Referring to Figure 1, we should aim to hold the risky asset when it generates risk-adjusted returns superior to cash, and reduce exposure if prospects are no longer attractive. Furthermore, if the expected returns are so high relative to the risk that the risk-return ratio warrants it, we could make use of leverage to allocate available even more efficiently. Under the assumption that leverage is available, we generalize that investors want a leveraged exposure if it is warranted by the risk-return ratio. The challenge at this point is to define this dynamic allocation such that we are always exposed according to the risk-return ratio of the risky asset.

## IMPROVING THE RISK-RETURN PROFILE

Assuming the risky asset follows a stochastic process and following the derivation as in (Giese, 2010), we obtain that the risky asset can be characterized by the expected long-term growth,  $g$ :

$$g = L\mu - (L - 1)r - \frac{1}{2}L(L - 1)\sigma^2$$

Where  $L$  is the optimal leverage proportion,  $\mu$  and  $\sigma$  are the expected rate of return and volatility of the risky asset respectively. This agrees with the findings of (Cheng & Madhavan, 2009, Despande, Mallick & Bhatia, 2009, and Lu, Wang & Zhang, 2009) but with the refinancing rate,  $r$ , included.

If we optimize this expression with respect to  $L$  by setting the first derivative to zero, we get the following:

$$L_{optimal} = \frac{\mu - r}{\sigma^2} + \frac{1}{2}$$

Having gone through a derivation from basic principles, we realize that the optimal allocation to a risky asset is proportional to the excess expected return and inversely proportional to the variance of the asset. Using this equation as a guide, and parametrizing the variables dynamically, we get a solution to our optimal allocation on a dynamic basis. We can now create a portfolio that would be allocated in such a fashion, and from this we create an index. An Optimal Leverage Index on the EURO STOXX 50 greatly outperforms the underlying index as shown on Figure 2.

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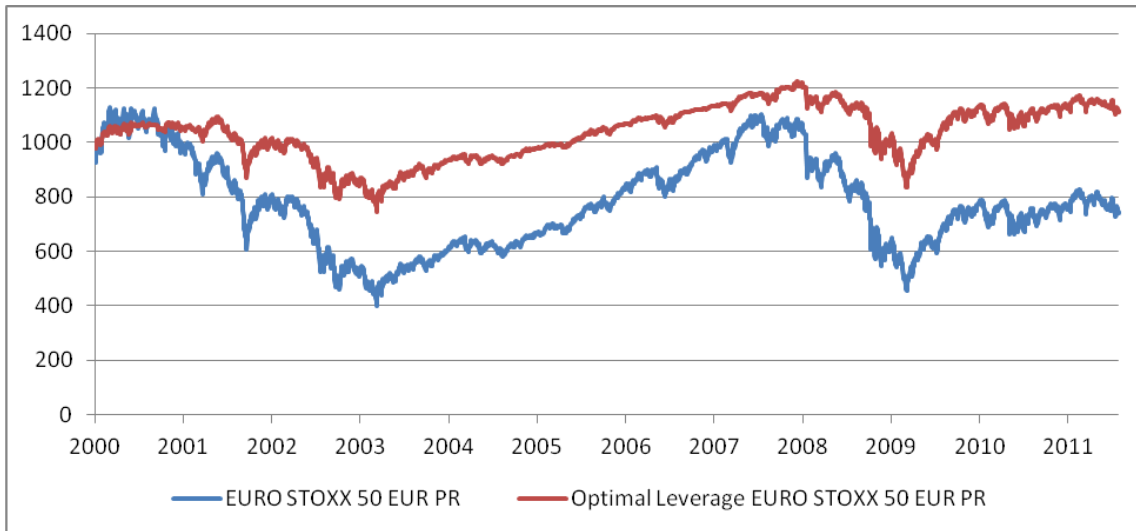


Figure 2: EURO STOXX 50 vs. Optimal Leverage

## OPTIMAL LEVERAGE

This concept is very flexible; it can be applied to any risky underlying, and investing in an optimal leverage version of it will have the underlying asset's liquidity. Such an Optimal Leverage index can be easily integrated within a structured product that would cater to investors who seek a superior risk-return profile and want to make use of leveraging capabilities. The maximum amount of leverage can be parametrized according to the investor's needs, and the concept can even be implemented as a long-short index, benefiting from short-selling capabilities to enhance the risk-return profile of the investor's portfolio. Having defined an optimal allocation formula, we can go even further than providing an optimal leverage index as we start looking into a whole new framework whereby simple market data is used to enhance the risk-return profile of available investment vehicles. Optimal leverage looks at investing an available portfolio optimally, but the same idea can be used to invest a risk budget optimally, in the case where the investor has capital requirements. Similarly, such an allocation formula can be used to optimize allocation between multiple risky assets.

## CONCLUSION

In conclusion, we have defined that there is the need for a dynamic portfolio management framework. We have derived that there is a mathematically derived optimal allocation proportion and that by using it, we are able to create a superior portfolio with a better risk-return profile. This can be used to allocate a portfolio optimally, but can also be extended to risk budgets, multiple risky assets and more.

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# OPTIMAL LEVERAGE: A DYNAMIC ALLOCATION APPROACH TO CAPTURING THE OPTIMAL RISK-RETURN RATIO

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