AUGUST 2017

STOXX® REFERENCE CALCULATIONS GUIDE

INNOVATIVE. GLOBAL. INDICES.

CONTENTS

1. INTRODUCTION TO THE STOXX INDEX GUIDES

The STOXX index guides are separated into the following sub-sets:

- » The **STOXX Calculation guide** provides a general overview of the calculation of the STOXX indices, the dissemination, the index formulas and adjustments due to corporate actions
- » The **STOXX Index Methodology guide** contains the index specific rules regarding the construction and derivation of the portfolio based indices, the individual component selection process and weighting schemes
- » The **STOXX Strategy guide** contains the formulas and description of all non-equity/strategy indices
- » The **STOXX Dividend Points Calculation guide** describes the dividend points products
- » The **STOXX Distribution Points Calculation guide** describes the distribution points products
- » The **STOXX ESG guide** contains the index specific rules regarding the construction and derivation of the ESG indices, the individual component selection process and weighting schemes
- » The **iSTOXX guide** contains the index specific rules regarding the construction and derivation of the iSTOXX indices, the individual component selection process and weighting schemes
- » The **STOXX Reference Rates guide** contains the rules and methodologies of the reference rate indices
- » The **STOXX Reference Calculations guide** provides a detailed view of definitions and formulas of the calculations as utilized in the reports, factsheets, indices and presentations produced by STOXX

All rule books are available for download on http://www.stoxx.com/indices/rulebooks.html

2. CHANGES TO THE GUIDE BOOK

2.1. HISTORY OF CHANGES TO THE STOXX REFERENCE CALCULATIONS GUIDE

- March 2014: First release of the guide
- July 2014: Addition of fundamentals calculations section
- December 2014: Addition of turnover calculation
- August 2017: Annualization factor set to 260 days

3. STATISTICAL CALCULATIONS

3.1. GENERAL DEFINITIONS

Any given time period can be divided in k equally-spaced intervals: $[t_m, t_{m+1}, \dots, t_{m+k}]$.

N is the nominal annualization factor of choice for such equally-spaced intervals, such that $N \cdot (t_z - t_{z-1})$ equals one nominal year. For instance, if the time interval length is one day, then $N = 260$ is used to annualize daily observations to a nominal year of 260 working days.

Accordingly, $k + 1$ price levels can be observed for a financial instrument: $[p_m, p_{m+1}, \dots, p_{m+k}]$.

3.2. RETURNS

Arithmetic and logarithmic returns can be calculated as shown below. Unless differently specified, arithmetic returns are the default calculation.

First, let the ratio of the price levels observed at two generic times t_i and t_j, with $t_m \leq t_i < t_j \leq t_{m+k}$, be expressed as:

(1) $$R_{i,j} = \frac{p_j}{p_i}$$

3.2.1. ARITHMETIC RETURNS

Then, the arithmetic return between time t_i and t_j is given by:

(2) $$r_{i,j} = R_{i,j} - 1$$

The actual return for a period of length k is then:

(3) $$r_{m,m+k} = R_{m,m+k} - 1$$

The corresponding annualized average return for a period of length k and with geometric compounding is given by:

(4) $$r_{k,ann} = \left(1 + r_{m,m+k}\right)^{\frac{N}{k}} - 1$$

3.2.2. LOGARITHMIC RETURNS

In case of log-returns, the actual and annualized returns are calculated respectively as:

(5) $$r_{m,m+k} = \ln R_{m,m+k}$$

and

(6) $$r_{k,ann} = r_{m,m+k} \cdot \frac{N}{k}$$

Note: For the sake of readability, the expression for a return time-series will be simplified in the following paragraphs according to the following notation:

(7) $$r = [r_1, \dots, r_k] = \left[r_{m,m+1}, \dots, r_{m+k-1,m+k}\right]$$

3.3. VARIANCE AND VOLATILITY

Variance and volatility are metrics used to represent how unpredictable the behavior of a statistical variable is. When the statistical variable is represented by a financial instrument's returns, they gauge the riskiness of that instrument.

Variance is usually calculated as a function of a financial instrument'sreturns deviation from their mean, i.e. including the drift term. However, the drift term can, under certain assumptions, be neglected: in this case, the mean value is set to zero.

Unless differently stated, variance is calculated including the drift term.

The returns used can be calculated either in arithmetic or logarithmic form.

3.3.1. VARIANCE WITH DRIFT

Given a time-series of k returns $r = [r_1, \dots, r_k]$, their (sample) variance is given by:

(8) $$\sigma^2(r) = \frac{1}{k-1} \cdot \sum_{i=1}^{k} (r_i - \bar{r})^2$$

where:

(9) $$\bar{r} = \frac{1}{k} \cdot \sum_{i=1}^{k} r_i$$

3.3.2. VARIANCE WITHOUT DRIFT

Simply, the mean return is ditched in the calculation of variance:

(10) $$\sigma^2(r) = \frac{1}{k-1} \cdot \sum_{i=1}^{k} r_i^2$$

Both variance measures can be annualized as:

(11) $$\sigma_{k,ann}^2(r) = \sigma^2(r) \cdot N$$

3.3.3. VOLATILITY

Once a measure of variance is calculated, the corresponding volatility is obtained by taking its square-root:

(12) $\sigma(r) = \sqrt{\sigma^2(r)}$ and $\sigma_{k,ann}(r) = \sqrt{\sigma_{k,ann}^2(r)}$

3. STATISTICAL CALCULATIONS

3.4. COVARIANCE

Covariance provides a measure of the co-movements of two statistical variables, or how the two variables move together: it shows the tendency in their linear relationship.

In broad terms, a positive (negative) covariance means that two variables exhibit a similar (different) behavior and tend to move in the same (different) direction(s); the larger the absolute value of covariance, the stronger the relationship.
A covariance of zero means that the observed variables tend to move in an uncoordinated way and expectations on the behavior of the one cannot be derived from the behavior of the other. The interpretation of the magnitude of the metric, however, is made difficult by the fact that covariance values are unbounded.

It is worthwhile to stress what covariance measures, i.e. the strength of the *linear approximation* of the *actual relationship* between two variables: while covariance is a useful aggregated indicator, a scatter plot of the variables can tell much about the nature of the relationship of the variables.

Similarly to variance, covariance can also be calculated including or excluding the drift term of both time-series involved.

Unless differently stated, covariance is calculated including the drift term.

3.4.1. COVARIANCE WITH DRIFT

Given two time-series of k returns $r = [r_1, \dots, r_k]$ for the reference financial instrument and $b = [b_1, \dots, b_k]$ for its benchmark, their sample covariance is given by:

(13) $$cov(r,b) = \frac{1}{k-1} \cdot \sum_{i=1}^{k} (r_i - \bar{r}) \cdot \left(b_i - \bar{b}\right)$$

3.4.2. COVARIANCE WITHOUT DRIFT

Both drift terms are removed:

(14) $$cov(r,b) = \frac{1}{k-1} \cdot \sum_{i=1}^{k} r_i \cdot b_i$$

Both covariance measures can be annualized as:

(15) $$cov_{k,ann}(r,b) = cov(r,b) \cdot N$$

3.5. CORRELATION

Correlation is a normalized representation of covariance and is bound within the range [-1,1]. The advantages over covariance are that a) correlation metric makes comparison among different variable pairs possible and b) the metric, being bounded, is easier to interpret.

Correlation inherits the caveats of covariance.

3. STATISTICAL CALCULATIONS

Given two time-series of k returns $r = [r_1, \dots, r_k]$ for the reference financial instrument and $b = [b_1, \dots, b_k]$ for its benchmark, their correlation is given by:

(16) $$\rho(r, b) = \frac{cov(r,b)}{\sigma(r) \cdot \sigma(b)}$$

Depending on the arguments' specifications, correlation can be calculated with or without drift, but it is not affected by the use of annualized values.

3.6. TRACKING ERROR

Tracking error gauges how closely a financial instrument tracks its benchmark: this is measured by the volatility of the return differential between the two.

Given two time-series of k returns $r = [r_1, \dots, r_k]$ for the reference financial instrument and $b = [b_1, \dots, b_k]$ for its benchmark, the tracking error is given by the volatility of an instrument's returns in excess of the benchmark's returns:

(17) $$TE(r, b) = \sigma\left(ER(r, b)\right)$$

where:

(18) $ER(r, b) = [ER_1(r_1, b_1), \dots, ER_k(r_k, b_k)]$ and $ER_i(r_i, b_i) = r_i - b_i$.

The annualized tracking error is given by:

(19) $$TE_{k,ann}(r, b) = \sigma_{k,ann}\left(ER(r, b)\right).$$

3.7. DRAWDOWN

Drawdown measures the magnitude of a financial instrument's loss since its last peak. The maximum drawdown, in turn, represents the largest loss suffered by a financial instrument in its history.

Like returns, drawdowns can be calculated in arithmetic or logarithmic form. Unless differently specified, arithmetic form is used.

Let the following ratio be defined:

(20) $$D_j = \frac{p_j}{\max\limits_{t \epsilon [t_i, t_j]} \{p_t\}}$$

3.7.1. ARITHMETIC DRAWDOWN

The arithmetic drawdown is given by:

(21) $$DD_j = D_j - 1$$

3.7.2. LOGARITHMIC DRAWDOWN

The logarithmic drawdown is given by:

(22) $$DD_j = \ln D_j$$

In both cases, the maximum drawdown is given by:

(23) $$MDD_j = \min_{t \leq j}\{DD_t\}$$

(24)

3.8. INFORMATION AND SHARPE RATIOS

3.8.1. INFORMATION RATIO

Information ratio measures excess return of a financial instrument over its benchmark, adjusted for the risk and it is obtained as the ratio of the average excess return to the tracking error of the two instruments.

Given two time-series of k returns $r = [r_1, \dots, r_k]$ for the reference financial instrument and $b = [b_1, \dots, b_k]$ for its benchmark, the information ratio is given by:

(25) $$IR(r,b) = \frac{\overline{ER}(r,b)}{TE(r,b)}$$

where:

(26) $$\overline{ER} = \frac{1}{k} \cdot \sum_{i=m+1}^{m+k} ER_i(r_i, b_i)$$

and the annualized IR is given by:

(27) $$IR_{k,ann}(r,b) = IR(r,b) \cdot \sqrt{N}$$

3.8.2. SHARPE RATIO

The Sharpe ratio is equivalent to the information ratio, where the benchmark is a risk-free security.

The Sharpe ratio is calculated using a risk-free rate time-series as benchmark. The actual and annualized Sharpe Ratio are calculated as:

(28) $$SR(r,rf) = IR(r,rf)$$

and

3. STATISTICAL CALCULATIONS

(29) $$SR_{k,ann}(r,rf) = IR_{k,ann}(r,rf)$$

with $rf = [rf_1, \dots, rf_k]$.

3.9. BETA

The beta of a financial instrument measures the sensitivity of a financial instrument's returns to the benchmark returns and can be seen as the volatility-adjusted correlation of the two. Equivalently, beta can be obtained as the slope of the Security Market Line in the Capital Asset Pricing Model (where the benchmark is the world portfolio).

Given two time-series of k returns $r = [r_1, \dots, r_k]$ for the reference financial instrument and $b = [b_1, \dots, b_k]$ for its benchmark, the beta of the instrument relative to the benchmark is given by:

(30) $$\beta(r,b) = \frac{cov(r,b)}{\sigma^2(b)} = \frac{\sigma(r)}{\sigma(b)} \cdot \rho(r,b)$$

Depending on the arguments' specifications, beta can be calculated with or without drift, but, as correlation, it is not affected by the use of annualized value.

4. FUNDAMENTALS CALCULATIONS

4.1. GENERAL DEFINITIONS

Parameter	Description
t	Index calculation time t
i	Index constituent i ($i = 1, \ldots, n$)
n	Number of constituents in $Index_{R,t}$ at time t
$p_{i,t}$	Close price of index constituent i at time t
$x_{i,t}$	Exchange rate from currency of constituent i to base currency of the index at time t
$q_{i,t}$	Number of shares of index constituent i at time t in the index, adjusted over time to account for any applicable corporate action, as defined in the STOXX Equity Calcualtion Guide: ➢ **Free-float market capitalization weighted index** The weighting scheme reflects the free-float market share of the index constituents: $q_{i,t} = s_{i,t} \cdot ff_{i,t} \cdot cf_{i,t}$ ➢ **Alternatively weighted index** The index constituents are weighted proportionally to a metric specific to the index concept, e.g. dividend yield or inverse of volatility: $q_{i,t} = wf_{i,t} \cdot cf_{i,t}$
$s_{i,t}$	Total number of shares of index constituent i at time t
$ff_{i,t}$	Free-float factor of index constituent i at time t
$cf_{i,t}$	Capping factor of index constituent i as deemed valid by STOXX at time t
$wf_{i,t}$	Number of shares of constituent i at time t, reflecting the specific weighting scheme adopted by the index
$ts_{i,t}$	Number of shares of constituent i traded at time t
$EPS_{i,t}$	Earnings per Share of index constituent i at time t
$BV_{i,t}$	Book Value of index constituent i at time t
$CF_{i,t}$	Cash Flow of index constituent i at time t
$Sales_{i,t}$	Revenues from Sales of index constituent i at time t

4. FUNDAMENTALS CALCULATIONS

$Index_{R,t}$	Close level of $Index$ in its variant R at time t. R can take values P, NR and GR respectively for Price, Net Return and Gross Return variant.
D_t	Divisor for $Index_{R,t}$ at time t

4.2. INDEX FUNDAMENTALS

4.2.1. DIVIDEND YIELD

For a Net or Gross Return index, the Dividend Yield is defined as its excess return as compared to the corresponding Price index over the chosen reference time period $[t_1, t_2]$:

(31) $$DY_{NR,[t_1,t_2]} = \frac{Index_{NR,t_2}}{Index_{NR,t_1}} - \frac{Index_{P,t_2}}{Index_{P,t_1}}$$

(32) $$DY_{GR,[t_1,t_2]} = \frac{Index_{GR,t_2}}{Index_{GR,t_1}} - \frac{Index_{P,t_2}}{Index_{P,t_1}}$$

4.2.2. PRICE/EARNINGS RATIO

The Price/Earnings ratio of an index is defined as the ratio between the aggregated market value and the aggregated Earnings per Share of its constituents:

(33) $$PE_t = \frac{\frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}}{\frac{\sum_{i=1}^{n} EPS_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}} = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{\sum_{i=1}^{n} EPS_{i,t} \cdot q_{i,t} \cdot x_{i,t}}$$

4.2.3. PRICE/BOOK VALUE RATIO

The Price/Book Value ratio of an index is defined as the ratio between the aggregated market value and the aggregated Book Value of its constituents:

(34) $$PB_t = \frac{\frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}}{\frac{\sum_{i=1}^{n} BV_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}} = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{\sum_{i=1}^{n} BV_{i,t} \cdot q_{i,t} \cdot x_{i,t}}$$

4.2.4. PRICE/CASHFLOW RATIO

The Price/Cash Flow ratio of an index is defined as the ratio between the aggregated market value and the aggregated Cash Flow of its constituents:

(35) $$PCF_t = \frac{\frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}}{\frac{\sum_{i=1}^{n} CF_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}} = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{\sum_{i=1}^{n} CF_{i,t} \cdot q_{i,t} \cdot x_{i,t}}$$

4. FUNDAMENTALS CALCULATIONS

4.2.5. PRICE/SALES RATIO

The Price/Sales ratio of an index is defined as the ratio between the aggregated market value and the aggregated Revenues of its constituents:

(36) $$PS_t = \frac{\frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}}{\frac{\sum_{i=1}^{n} Sales_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{D_t}} = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t} \cdot x_{i,t}}{\sum_{i=1}^{n} Sales_{i,t} \cdot q_{i,t} \cdot x_{i,t}}$$

4.3. SECURITY AVERAGE DAILY TRADED VALUE (ADTV)

The Average Daily Traded Value represents the value of trades executed on an average day in a reference time period for a certain security.

Security ADTVs are processed by country.
For each security i belonging to the observed country c, all prices and traded quantities available during the selected calendar period are taken: this may lead to a different number N_i of total records per each security i.
To simplify the identification of non-trading days, for each security the number $NrNull_i$ of null records is counted; then an adjustment factor for non-trading days is calculated for each country c as:

(37) $$NTD\_adj_c = \min_{i \in c}\{NrNull_i\}$$

Consequently, the ADTV for security i belonging to country c is calculated as:

(38) $$ADTV_{i \in c, \left[t_1, t_{N_i}\right]} = \frac{\sum_{t=t_1}^{t_{N_i}} p_{i,t} \cdot ts_{i,t} \cdot x_{i,t}}{N_i - NTD\_adj_{c \ni i}}$$

4.4. TURNOVER

The turnover of an index indicates what portion of it is bought or sold over a certain period, following rebalancing events: it can thus be seen as a gauge of the amount of trading needed to replicate that index.

STOXX provides an annualized one-way turnover measure, based on quarterly data, i.e. on the quarterly Review events.
For a given index, the turnover for a given Review event is calculated as follows:

1. take the index composition list valid for the close of the index Review Implementation date (quarter $Q-1$)

2. take the index composition list valid for the open of the index Review Effective date (quarter Q)

3. create a pool of the securities from both composition lists

4. FUNDAMENTALS CALCULATIONS

4. for each security i, calculate the weight change – in absolute terms – between the two compositions:

(39) $$TO_{i,Q} = \frac{|w_{i,open,Q} - w_{i,close,Q-1}|}{2}$$

5. calculate the index turnover as sum of all turnovers of the n securities in the index:

(40) $$TO_{index,Q} = \sum_{i=1}^{n} TO_{i,Q}$$

6. the annualized turnover is then given by:

(41) $$TO_{index,ann} = \frac{4}{q'} \cdot \sum_{q=0}^{q'} TO_{index,Q-q}$$

where q' is the number of preceding quarterly data available and is capped to 3.

If $\mathrm{q}' = 0$, no annualized turnover is provided.