Research Report:

Risk Control Strategies

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Executive Summary

Advanced index concepts which try to optimize the risk/return profile and protect investors from losses have recently gained popularity. As a new approach the so-called target risk or risk control strategy defines ex-ante a risk level, where portfolio's assets are regularly reallocated between a risk-free and risky asset so that overall portfolio's volatility matches the predefined target risk. This paper empirically analyzes target risk strategies of EURO STOXX 50 stock market indices in the context of alternative optimization approaches such as minimum-variance-based and equally-weighted indices. Empirical evidence shows that target risk strategies are more risk-return efficient than the capital-weighted benchmark. Compared to a minimum variance the risk control strategies exhibit by far more stable risk levels over time and clearly outperform the naïve portfolio.

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1 Introduction

The last decade has seen a massive rise in index-linked, passive investments. Meanwhile they account for more than half of all institutional investments. In particular exchange traded funds (ETFs) which replicate common indices have become very popular with retail investors. Reasons for that are transparency, efficiency, and the well-known inability of active fund management to beat their respective benchmark index on average; at least in the blue chip universe and after costs.[1]

In addition, the financial crisis over the last years has raised the risk awareness of investors. Before the crisis the focus was on the cost efficient and transparent tracking of indices which where usually weighted according to the market capitalization of their components. Advanced, rules-based index concepts have recently gained market shares since they offer higher transparency to investors. They still represent the performance of a certain market segment but try to protect investors from severe market downturns and/or improve the risk return profile. In particular minimum variance and equally weighted portfolios, which are both well studied in academic literature, are now implemented in portfolio management.

The latest approach are risk controlled (or target risk) investment strategies. The intuition behind is to shift the allocation from risky assets to the money market during volatile market times and vice versa. This builds on the observation of a negative correlation between the return and volatility of equity markets, i.e. very volatile periods are more often associated with negative returns. The weighting rule is endogenously given by the volatility of the underlying equity market index and exogenously determined by the pre-defined target volatility of the overall portfolio. During times of high volatility the strategy invests partly in the money market to protect investors, while during periods of low volatility the funds are completely invested in the equity index. Depending on the level of target volatility this may

[1] Barras, Scaillat, and Wermers (2010) show that over a period of 30 years and over more than 2,000 active funds less than one percent exhibits positive alphas after controlling for pure luck.

also lead to a leveraged investment in the equity index. As a result, a target risk strategy should participate more in rising and be less affected from falling markets than a pure investment in the respective market index. Beside the return dimension, target risk approaches offer a controlled risk profile. For instance, in life cycle investing the target risk level can be systematically lowered as time passes by. In this research report we empirically analyze the properties of target risk or risk control approaches compared to a pure equity index investment. We additionally place them into the context of alternative optimization techniques such as the minimum variance or equally-weighted portfolios.

Starting with the seminal paper of Markowitz (1952) considerable effort has been made to utilize mean-variance optimization in portfolio management. Typically these optimization models produce extreme fluctuation in the portfolio weights which react very sensitive to even small changes in the return parameter. Because the estimation error from using past returns to predict future returns usually erodes the gain from optimal diversification strategies, research in portfolio management has focused on minimum variance portfolios. It constitutes the portfolio on the efficient frontier that requires no return forecasts. Because variances and covariances are the only parameters needed to minimize portfolios risk, estimated portfolio weights are more stable over time than those of mean-variance portfolio.

Naïve or equally-weighted portfolios are a counter approach to the sophisticated world of portfolio optimization. All assets are weighted equally and the portfolio weights are simply rebalanced according to the number of assets. Thus, corner solutions and concentration risks can be avoided. Although the naïve strategy attracted less academic research than mean-variance optimization, in testing minimum-variance strategies it replaced capital-weighted indices as more 'challenging to beat' the benchmark in recent studies as both usually outperform the common market indices in their risk-return dimension.[2]

[2] The development of new portfolio optimization strategies, in particular minimum-variance approaches vs. equally-weighted strategies remains a horserace-like challenge. DeMiguel, Garlappi, and Uppal (2009)

In this research report we use the EURO STOXX 50 from January 1992 to December 2011 as equity index and relevant universe for the minimum variance and equally-weighted portfolio strategies. We apply different rolling windows to adjust our portfolio weights (i.e., in the money market and the equity index as well as the individual weights of our index components). The success of the target risk strategy critically depends on the ability to predict the volatility of the underlying index for the holding period. We use three volatility measures: sample (unconditional) volatility, conditional volatility, and implied volatility. While conditional volatility derived from GARCH models accounts for time-varying volatility and volatility clustering, implied volatility has the property of a forward looking measure.

Our results show that the risk control strategies clearly outperform the capitalization weighted index with a Sharpe ratio more than twice as high (for similar risk levels). The outperformance comes from the timing ability to leverage and deleverage depending on the market condition. Compared to the minimum variance approach the realized volatility of the target risk strategy stays in a narrow band over time, whereas it heavily fluctuates for the minimum variance portfolio. It also outperforms the naïve portfolio strategy. As only the index needs to be traded for the risk control strategy, transaction costs should not erode the potential gains.

The research report is organized as follows: Section 2 outlines our empirical design and dataset. In section 3 we represent our results. Section 4 gives concluding remarks.

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compare numerous sample-based mean-variance strategies to a naïve portfolio strategy. They conclude that none of the sophisticated strategies is consistently better in terms of the Sharpe ratio. In contrast, e.g., Fletcher (2009) among others report consistently higher Sharpe ratios for their minimum variance models than for the equally-weighted benchmark. Tu and Zhou (2011) use the same dataset as DeMiguel, Garlappi, and Uppal (2009). They document an improvement over the naïve strategy for their estimators which combine the naïve rule with sophisticated strategies.

2 Methodology and Data

We analyze risk control strategies as pure rule-based investment strategies. They combine an investment in equity with the money market, which we assume to be risk-free. The equity market is represented by a liquid equity index which allows for convenient replication and cost-efficient rebalancing. The basic idea is that the investor sets a target volatility which is matched by allocating the funds to the risky asset and the secure money market. If the (expected) risk of the risky asset is above the target, money is shifted to the riskless money market. In contrast, if the risk is below the predefined level the portfolio risk is increased by taking a leveraged position.

The rationale behind is that volatile times in the equity market are more often associated with negative than positive returns which can be measured as negative correlation between returns and volatility.[3] Figure 1 illustrates the relationship between volatility and return for the EURO STOXX 50. Both variables are strongly negative correlated as can be seen by the negative slope of the second order polynomial regression line (correlation -0.37).

The risk control strategy weights the investment in the risky equity index and the money market according to the expected volatility for the next holding period and the predefined target volatility, i.e. the investment in the money market is increased if volatile times are expected to be ahead. As we assume the investment in the money market to be risk-free, the investment in the equity market is defined as fraction x of total investments

$$x = \frac{s_{target}}{s_{expected}},$$

where s_{target} is the target volatility and $s_{expected}$ is the expected volatility of the equity index for the pre-defined allocation period.

[3] For a theoretical framework in the context of target risk strategies see Giese (2011).

Figure 1: Risk and Return Relationship

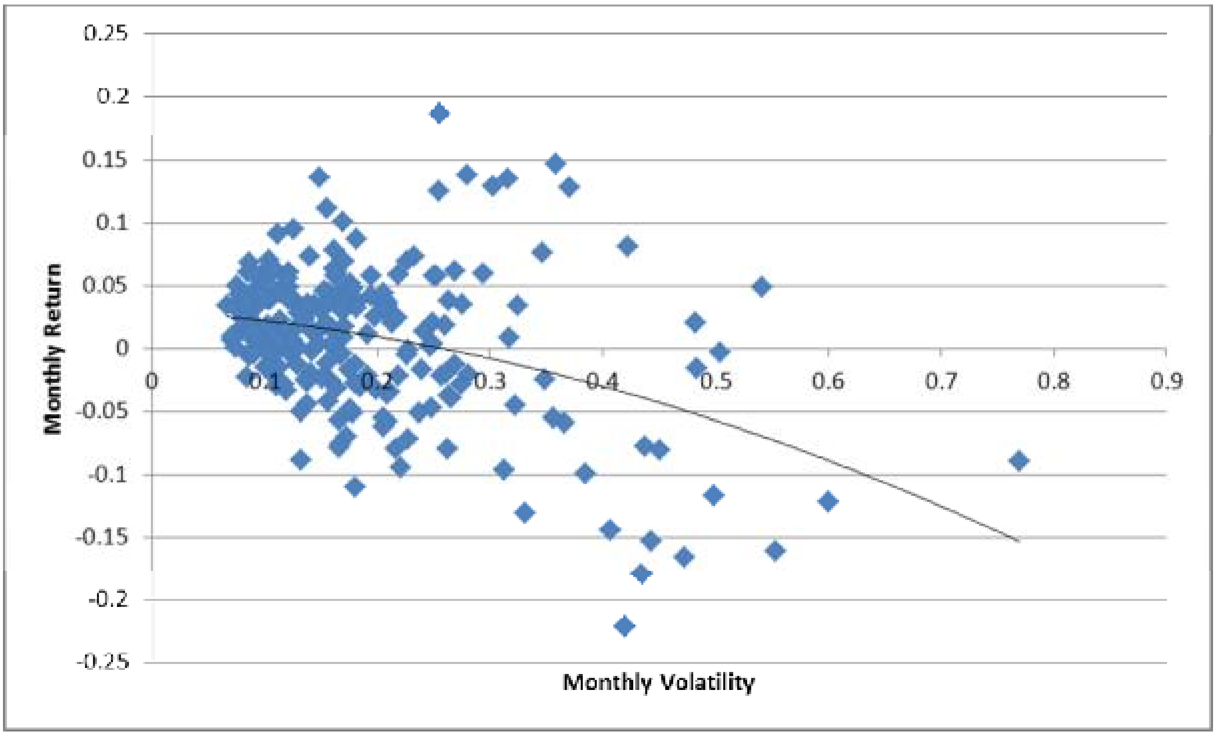

Figure 1 shows the relationship between risk and return for the EURO STOXX from January 1992 to December 2011. The monthly returns and volatilities are annualized. We run a second order polynomial regression as depicted by the line.

The success of the target risk strategy crucially depends on the estimate of the volatility of the risky asset for the holding period. We apply three measures, namely the sample volatility of the previous holding period and the implied volatility derived from option prices.[4]

As a third estimate for future volatility, we derive volatility forecasts from a GARCH(1,1) model (Bollerslev, 1986):

$$r_t = m + e_t$$
$$e_t = \sqrt{h_t} x_t$$
$$x_t \quad N(0,1)$$

[4] Our empirical study focuses on the EURO STOXX as equity index and its relevant universe for alternative investments strategies. But as a proxy we use the VDAX instead of the VSTOXX as the latter one is only available since 1999. However, note that for the period 1998 to 2011, the correlation coefficient (Pearson) between the two is approximately 99%.

$$h_t = w + a e_{t-1}^2 + b h_{t-1}$$

We thereby predict the volatility based on its long-term average, the forecasted volatility from last period and volatility observed in previous periods. This can be interpreted as adaptive learning or Bayesian updating. It allows us to explicitly to account for volatility clustering.

We report our results for target volatility levels ranging from 5% to 40%. Besides one month and three months reallocation periods we apply a dynamic rule where we reallocate the assets whenever the portfolio weights (i.e. index and money market) would change by more than five percentage points based on a new current volatility estimate (5%-rule).[5]

For the minimum variance portfolio, we apply a rolling window approach, were we calculate the variance-covariance matrix based on data for one year (250 trading days) to determine the individual portfolio weights for the following holding period. We apply out-of-sample reallocation intervals of one month and three months.[6] We do not allow for short sales and place a 10% cap on individual stocks.[7] The historical estimator is the most common and simplest technique. As a maximum-likelihood estimator of the portfolio weights it minimizes the portfolio variance based on the historical variance-covariance matrix $\hat{\Omega}_H$:

$$\hat{\Omega}_H = \frac{1}{T-1}\sum_{i=1}^{T}(r_t - \bar{r})(r_t - \bar{r}) \tag{3.1}$$

where T is the sample size, r_t is a $N x 1$ vector of stock returns in period t, and $\bar{r}$ is the average of these return vectors. In contrast, the equally-weighted portfolio does not need any estimation technique. The weights in the naïve strategy are 1/*N*. It is rebalanced with respect to our target risk and according to the minimum variance strategies.

[5] For a discussion of daily vs. monthly returns for rebalancing portfolios see Trainor (2011).

[6] In the case of index entries we re-estimate the variance-covariance matrix with the price history of the new index constituent. If no sufficient history is available we keep the weight by the index weight fix until sufficient historical information is available.

[7] Deutsche Börse also uses a 10% cap in its index calculation.

The investment universe for our empirical analysis is the EURO STOXX 50 in its respective composition from January 1992 to December 2011. We collected all daily price data as well as dividends, stock splits etc. from Thompson Reuters Datastream. Our dataset includes all previous and actual constituents at the time of December 2011. This leaves us with 117 time series for the EURO STOXX 50 constituents. Calculations are based on log returns. As proxy for the risk-free interest rate we use the Euro Over Night Index Average (EONIA). We also report historical 95% VAR (calculated as $VaR_i = m_{i,t} + zs_{i,t}$), the portfolio turnover per reallocation to evaluate transaction costs and the average investment in the money market (or leverage) for the different target risk control portfolios.

3 Empirical Results

Tables 1 to 3 report the descriptive statistics and performance ratios of the different investment strategies for the monthly and quarterly reallocation periods, as well as 5%-ruleover the entire sample period. Figures 1 and 2 display the return and risk dimension over time. In the discussion of the results, we first focus on the risk control strategy with a one-month reallocation period and a twenty percent risk target based on the volatility of the previous period.

Table 1: Risk and Return Performance for Monthly Reallocations

Investments Strategy	Mean	Std. Dev.	Sharpe Ratio	Value at Risk	Turnover	Leverage
Index	7.13%	22.46%	13.92%	29.81%	0.00%	-
Equally-Weighted	9.57%	22.61%	24.64%	27.62%	6.26%	-
Minimum Variance	9.96%	17.01%	35.04%	18.01%	30.58%	-
Risk Control Index 05%	4.78%	5.01%	15.60%	3.46%	9.69%	29.50%
Risk Control Index 10%	6.82%	10.06%	28.04%	9.73%	19.37%	59.01%
Risk Control Index 15%	8.64%	15.17%	30.56%	16.32%	29.65%	88.51%
Risk Control Index 20%	10.21%	20.34%	30.52%	23.25%	40.70%	118.02%
Risk Control Index 25%	11.52%	25.58%	29.39%	30.56%	52.60%	147.52%
Risk Control Index 30%	12.56%	30.92%	27.67%	38.31%	65.42%	177.03%
Risk Control Index 40%	13.75%	41.97%	23.23%	55.28%	93.90%	236.03%
Risk Control Impl. Vola. 05%	4.17%	4.16%	4.11%	2.67%	5.71%	24.05%
Risk Control Impl. Vola. 10%	5.65%	8.34%	19.80%	8.07%	11.27%	48.09%
Risk Control Impl. Vola. 15%	6.96%	12.56%	23.59%	13.70%	17.23%	72.14%
Risk Control Impl. Vola. 20%	8.10%	16.82%	24.35%	19.57%	23.93%	96.18%
Risk Control Impl. Vola. 25%	9.04%	21.13%	23.86%	25.71%	31.28%	120.23%
Risk Control Impl. Vola. 30%	9.79%	25.48%	22.71%	32.13%	39.23%	144.28%
Risk Control Impl. Vola. 40%	10.64%	34.38%	19.32%	45.91%	57.19%	192.37%
Risk Control GARCH 05%	5.80%	7.86%	22.84%	7.13%	12.77%	45.51%
Risk Control GARCH 10%	8.51%	15.84%	28.46%	17.54%	26.55%	91.02%
Risk Control GARCH 15%	10.60%	23.97%	27.56%	28.82%	42.34%	136.52%
Risk Control GARCH 20%	12.03%	32.32%	24.84%	41.13%	60.41%	182.03%
Risk Control GARCH 25%	12.72%	40.97%	21.29%	54.67%	81.15%	227.54%
Risk Control GARCH 30%	12.63%	50.05%	17.25%	69.70%	104.92%	273.05%
Risk Control GARCH 40%	9.87%	71.01%	8.27%	106.93%	163.82%	364.07%

Table 1 summarizes the results for the different investment strategies for monthly reallocations. We have a total of 247 reallocations in our sample period ranging from January 1992 to November 2011. The mean and the standard deviation are annualized. The value-at-risk is measured at the empirical 95% confidence level. Turnover measures the average turnover per reallocation. Leverage measures the average allocation to the index and the money market (i.e., a leverage of 80% indicates that 80% of the total funds are invested in the index and 20% in the money market).

Table 1 shows that all three rule based strategies (equally weighted, minimum variance, and RiskControlIndex 20%) outperform the in capitalization weighted index in the return dimension by more than two percent per year. The Risk Control Index 20% offers the highest return with more than three percentage points above the index. The average standard deviation is the lowest for the minimum variance strategy which translates into the highest Sharpe ratio. With an average annualized standard deviation of 20.34% the risk control strategy is very close to the predefined target of 20%. It offers the second highest Sharpe ratio which is still about 2.5 times higher than that of the index.

Table 2: Risk and Return Performance for Quarterly Reallocations

Investment Strategy	Mean	Std. Dev.	Sharpe Ratio	Value at Risk	Turnover	Leverage
Index	7.13%	22.46%	13.92%	29.81%	0.00%	-
Equally-Weighted	9.30%	22.46%	23.60%	27.64%	12.45%	-
Minimum Variance	9.12%	17.32%	29.59%	19.36%	61.88%	-
Risk Control Index 05%	4.89%	5.28%	16.93%	3.79%	16.48%	29.49%
Risk Control Index 10%	6.96%	10.65%	27.84%	10.55%	32.29%	58.97%
Risk Control Index 15%	8.73%	16.14%	29.28%	17.83%	48.88%	88.46%
Risk Control Index 20%	10.16%	21.83%	28.24%	25.74%	66.93%	117.94%
Risk Control Index 25%	11.25%	27.77%	26.12%	34.42%	86.40%	147.43%
Risk Control Index 30%	11.97%	34.08%	23.37%	44.09%	107.19%	176.91%
Risk Control Index 40%	12.11%	48.69%	16.66%	67.98%	157.08%	235.88%
Risk Control Impl. Vola. 05%	4.39%	4.36%	8.84%	2.79%	9.74%	24.00%
Risk Control Impl. Vola. 10%	6.04%	8.79%	23.15%	8.43%	18.26%	48.00%
Risk Control Impl. Vola. 15%	7.47%	13.31%	26.07%	14.42%	27.38%	71.99%
Risk Control Impl. Vola. 20%	8.68%	17.95%	26.08%	20.84%	37.60%	95.99%
Risk Control Impl. Vola. 25%	9.65%	22.73%	24.86%	27.74%	48.81%	119.99%
Risk Control Impl. Vola. 30%	10.37%	27.72%	22.98%	35.23%	61.20%	143.99%
Risk Control Impl. Vola. 40%	10.99%	38.59%	18.10%	52.49%	89.83%	191.98%
Risk Control GARCH 05%	6.20%	8.47%	26.02%	7.73%	18.13%	46.38%
Risk Control GARCH 10%	9.18%	17.19%	30.12%	19.10%	37.27%	92.75%
Risk Control GARCH 15%	11.38%	26.34%	28.02%	31.94%	61.78%	139.13%
Risk Control GARCH 20%	12.73%	36.21%	24.10%	46.83%	90.47%	185.51%
Risk Control GARCH 25%	13.10%	47.36%	19.22%	64.79%	124.68%	231.89%
Risk Control GARCH 30%	12.35%	61.24%	13.63%	88.38%	166.68%	278.26%
Risk Control GARCH 40%	6.40%	249.25%	0.96%	403.58%	283.21%	371.02%

Table 2 summarizes the results for the different investment strategies for quartely reallocations. We have a total of 283 reallocations in our sample period ranging from January 1992 to November 2011. The mean and the standard deviation are annualized. The value-at-risk is measured at the empirical 95% confidence level. Turnover measures the average turnover per reallocation. Leverage measures the average allocation to the index and the money market (i.e., a leverage of 80% indicates that 80% of the total funds are invested in the index and 20% in the money market).

The results for the 95%-Value-at-Risk are similar to the standard deviation. It is the lowest the minimum variance portfolio followed by the Risk Control Index 20%. It is about the same for the equally-weighted and capital-weighted approaches. The results for the risk control index based on the implied volatility or the GARCH(1,1) model are generally worse.[8] But they still outperform the capitalization weighted index for similar risk levels. The realized volatility of the target risk index based on the implied volatility is on average lower than the target level and higher for our GARCH(1,1) model.

[8] Giese (2010) proposes to use implied volatilities. Our results do not support this recommendation.

Table 3: Risk and Return Performance for the 5%-Rule

Methodology	Mean	Std. Dev.	Sharpe Ratio	Value at Risk	Turnover	Leverage
Risk Control Index 05%	4.80%	4.86%	16.50%	3.19%	0.77%	35.33%
Risk Control Index 10%	6.88%	9.79%	29.38%	9.22%	1.62%	69.70%
Risk Control Index 15%	8.58%	14.67%	31.21%	15.55%	2.95%	103.88%
Risk Control Index 20%	10.04%	19.54%	30.92%	22.10%	4.17%	138.30%
Risk Control Index 25%	11.49%	24.43%	30.64%	28.70%	5.20%	173.12%
Risk Control Index 30%	12.64%	29.32%	29.45%	35.60%	7.05%	206.66%
Risk Control Index 40%	14.06%	39.14%	25.71%	50.31%	11.35%	270.89%
Risk Control Impl. Vola. 05%	4.00%	4.04%	n.a.	2.64%	0.83%	26.01%
Risk Control Impl. Vola. 10%	5.53%	8.05%	18.99%	7.71%	1.34%	53.05%
Risk Control Impl. Vola. 15%	6.83%	12.10%	23.36%	13.07%	2.86%	77.95%
Risk Control Impl. Vola. 20%	7.95%	16.08%	24.59%	18.50%	4.22%	103.62%
Risk Control Impl. Vola. 25%	8.97%	20.11%	24.73%	24.10%	6.20%	127.81%
Risk Control Impl. Vola. 30%	9.87%	24.12%	24.33%	29.81%	8.18%	152.90%
Risk Control Impl. Vola. 40%	10.85%	32.18%	21.27%	42.09%	11.15%	200.42%
Risk Control GARCH 05%	5.94%	7.52%	25.85%	6.42%	2.14%	48.61%
Risk Control GARCH 10%	8.92%	15.09%	32.61%	15.90%	5.67%	94.07%
Risk Control GARCH 15%	11.25%	22.76%	31.86%	26.18%	9.66%	142.22%
Risk Control GARCH 20%	12.89%	30.43%	29.20%	37.17%	14.40%	186.68%
Risk Control GARCH 25%	13.93%	38.08%	26.07%	48.71%	18.56%	232.21%
Risk Control GARCH 30%	14.25%	45.72%	22.42%	60.95%	22.75%	276.53%
Risk Control GARCH 40%	12.73%	61.05%	14.31%	87.69%	32.44%	365.94%

Table 2 summarizes the results for the different investment strategies for the 5%-rule. The mean and the standard deviation are annualized. The value-at-risk is measured at the empirical 95% confidence level. Turnover measures the average turnover per reallocation. Leverage measures the average allocation to the index and the money market (i.e., a leverage of 80% indicates that 80% of the total funds are invested in the index and 20% in the money market).

Table 2 summarizes the results for the quarterly reallocation. The Sharpe ratios for the risk control indices based on the volatility of the previous period as well as for the minimum variance strategy slightly deteriorate. It stays about the same for the naïve portfolio. The performance of the risk control approach using the GARCH(1,1) model noticeable improves showing now the highest Sharpe ratio of all allocations for the Risk Control GARCH 20% with a realized volatility of 17%. Table 3 reports the findings for the 5%-rule. They are very similar to the monthly reallocations for the risk control indices based on previous volatilities and the implied volatility. The GARCH(1,1) works best in combination with the 5%-rule.

Figure 2: Price Development of the Different Investment Strategies

Figure 2 shows the performance of the capitalization weighted index, its equally weighted version, the minimum variance portfolio and the RiskControlIndex 20% over time. The starting value is set at 100.

Figure 2 shows the performance of the capitalization weighted index, the equally-weighted, and the minimum variance portfolio, as well as the Risk Control Index 20%. The target risk index clearly outperforms the alternatives in booming markets due to its leverage. Despite its risk cap by construction it relatively looses more in falling markets although the overall performance is still considerably better than in particular for the index.

As seen in Figure 3 the target risk strategy provides a relative stable volatility level over time. The volatility of the index and its equally-weighted version is about half of the target risk index from 1993 to 1996 and 2004 to 2007, it more than doubles in 2002/3 and 2008/9. The minimum variance strategy clearly reduces the volatility when compared to the index but the “variability” of the volatility by far higher than that of the target risk strategy.

Figure 3: Risk Development of the Different Investment Strategies

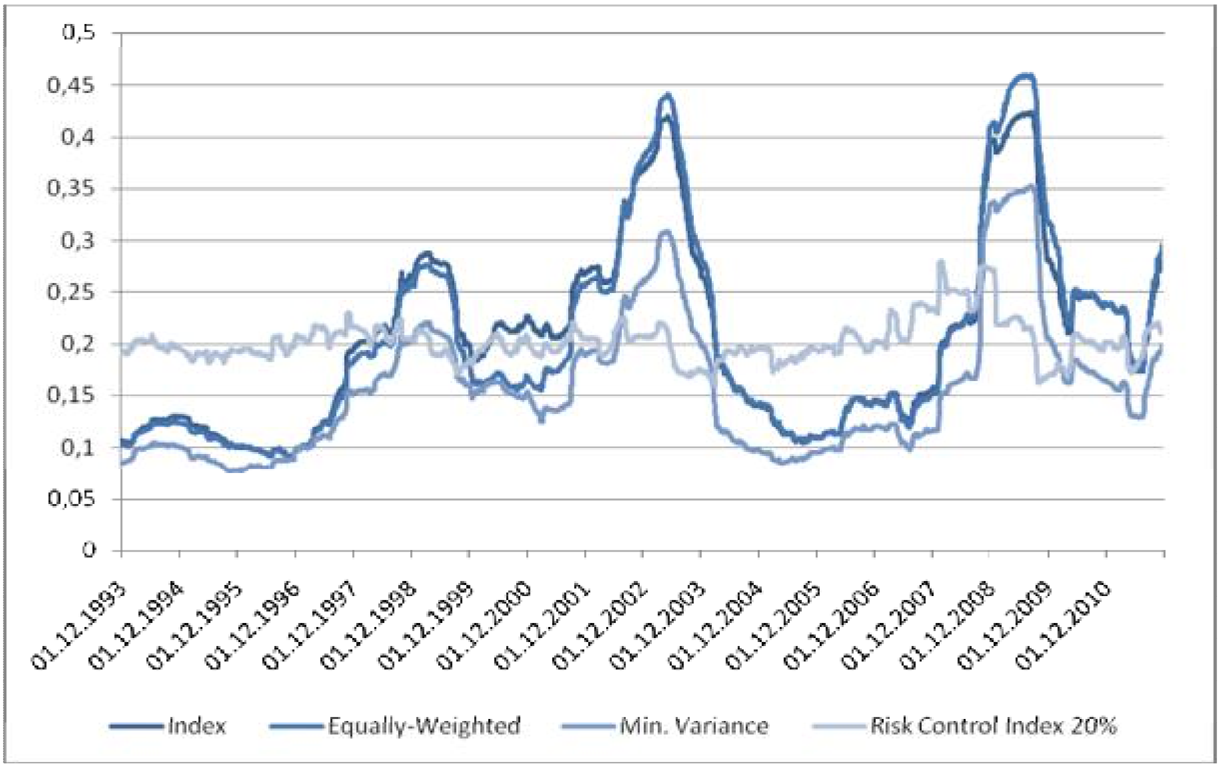

Figure 3 shows the annualized standard deviation of the of the capitalization weighted index, its equally weighted version, the minimum variance portfolio and the RiskControlIndex 20% with monthly reallocation.

Figure 4 illustrates the mismatch between the predefined risk level of 20% and the actually realized volatility. Except for the year 2008 where it jumps to 28% the (annualized) volatility remains within a bandwidth from 17% to 23%.

Figure 5 shows the leverage of the Risk Control Index 20% versus the realized volatility of the EURO STOXX 50. The leverage over time is inverse to the volatility which confirms the idea of the target risk control strategies. During times of very low market volatilities the leverage even reaches a level of three (i.e., 100% equity plus 200% short in the money market). In periods of a high market risk up to 30% of total funds are invested in the money market.

Figure 4: Volatility Mismatch

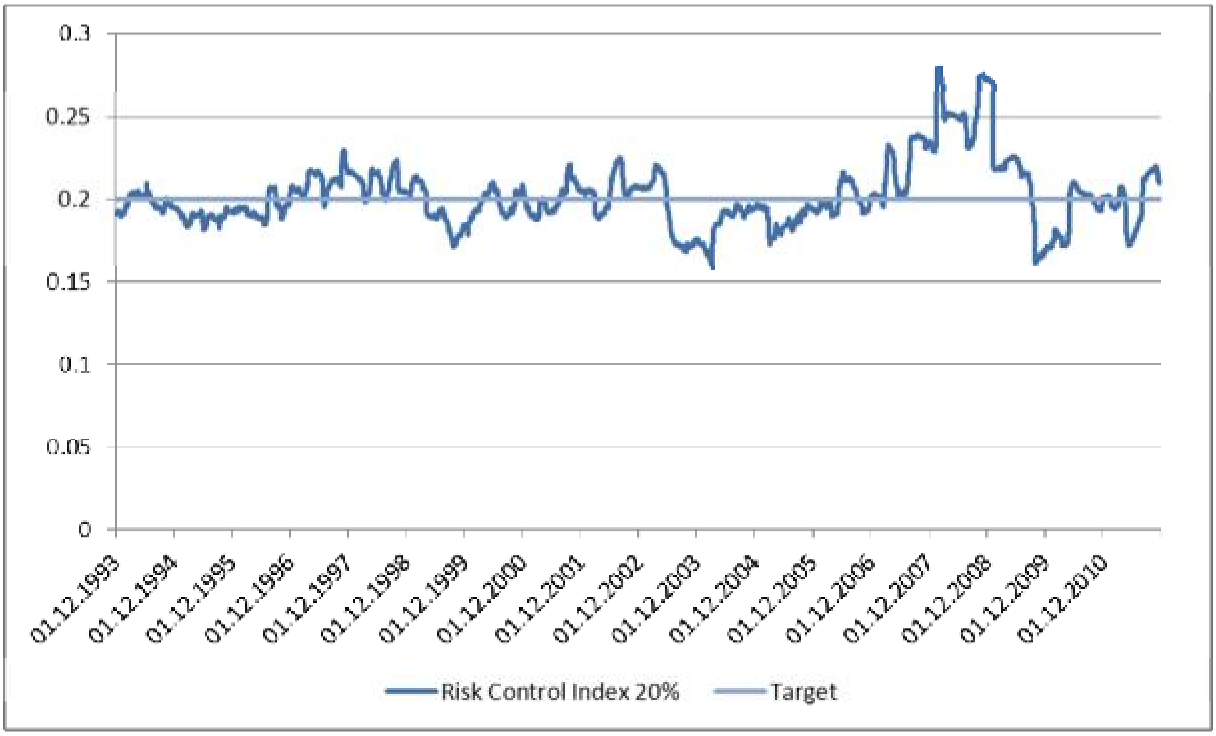

Figure 4 shows the mismatch between the target risk strategy with a predefined risk level of 20% and the realized volatility (annualized values).

Figure 5: Market Risk and Leverage

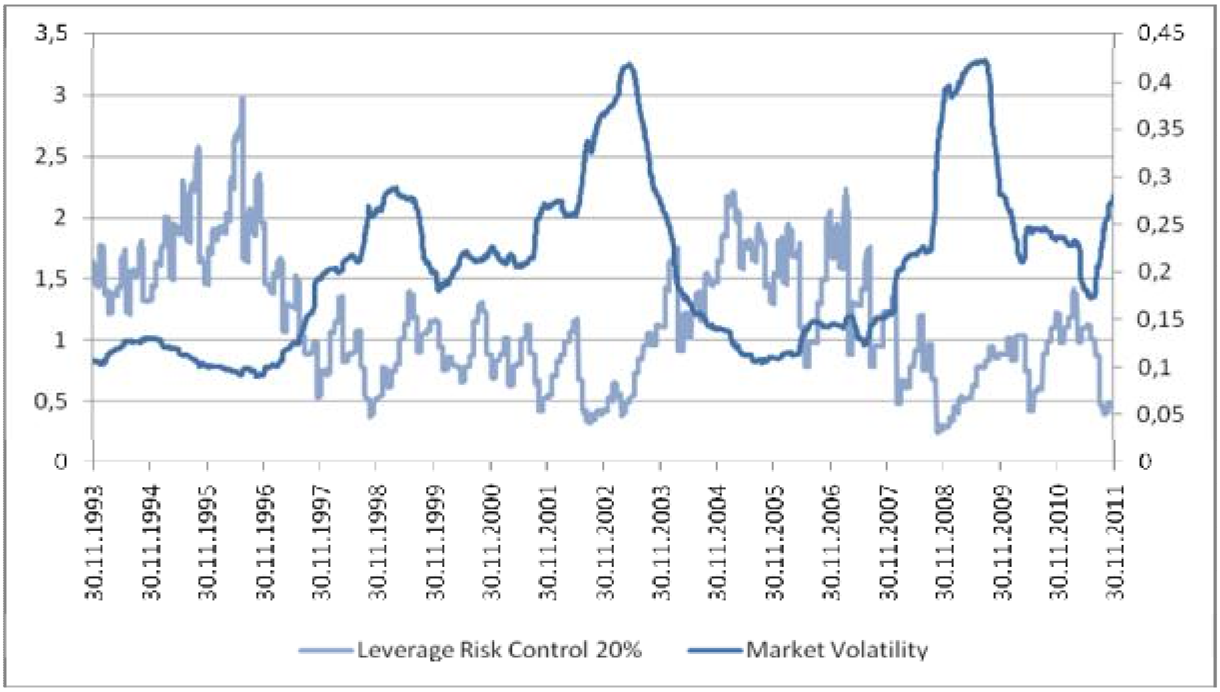

Figure 5 shows the leverage of the Risk Control Index 20% versus the realized volatility of the EURO STOXX 50.

Figure 6 shows the Sharpe ratios for the three different risk control strategies (based on sample, implied, and conditional volatility). The Sharpe ratios are inverse parabola-shaped. Thus, there is (ex post) an optimal target risk level which maximizes the risk-return efficiency. The reason for that is the path dependency of leveraged returns.[9] A higher leverage exploits potentially higher returns of the risky asset compared to the riskless money market, but a higher leverage also leads to increased volatility losses.[10]

Figure 6: Sharpe-Ratios for Different Risk Control Strategies

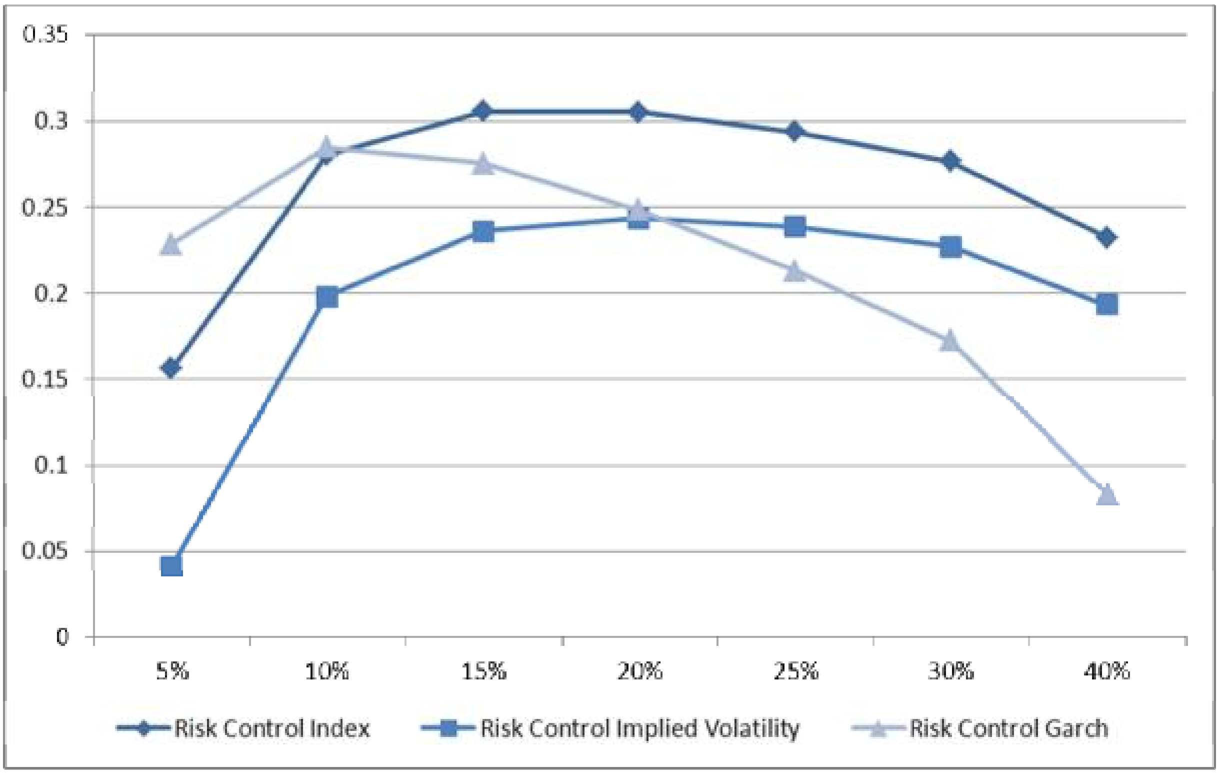

Figure 6 displays the Sharpe ratios for the different risk control strategies depending on the target risk with monthly reallocation.

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[9] Assume an underlying asset starts at 100 and moves in the first period by -30% to 70 and the following period by +42,6% back to 100. A leveraged fund with a leverage of two (and no refinancing costs) moves by -60% to 40 and by 85,2% to a final value of 74,1. Charupat and Miu (2011) and Lu, Wang, and Zhang study empirically study the behaviour of leveraged ETFs with daily rebalancing. Also see Chang and Madhavan (2009) and Jarrow (2010).

[10] The volatility losses are proportional to the squared leverage. For a comprehensive mathematical derivation see Giese (2010, 2011).

Empirically the optimal risk is around 15% to 20% (pre-defined target risk as well as sample volatility, Risk Control Index) which is slightly below the standard deviation of the index itself (about 22%).[11] The average leverage is about 0.9 (15% target risk level) and 1.2 (20% target risk level). This underlines that strong outperformance of the risk control indices in the risk-return efficiency compared to the index comes from the timing ability, i.e. to leverage and deleverage depending on the market condition. High risk strategies like a target risk of 40% or higher do not pay out due to volatility losses.

So far the analysis ignored the costs associated with the different investment strategies. The regular rebalancing of portfolios will lead to transaction costs. To capture potential transaction costs we calculated the average portfolio turnover per reallocation which is reported in column six in the Tables 1 to 3.[12] The average turnover for the Risk Control Index 20% is about 40% per reallocation and 30% for the minimum variance strategy for monthly reallocation. It is only about 6% for the equally weighted strategy. The average turnover per reallocation is higher for the quarterly reallocations but lower if annualized. The crucial difference between the risk control approach and the alternatives is that for the latter ones only the index needs to be traded which can be done in a very cost efficient way via ETFs or futures. In contrast, for the other alternatives the underlying stocks have to be traded.

4 Summary

Target risk or risk control approaches can be classified as advanced, rule based index strategies. The idea behind is to shift the allocation from risky assets to the money market

[11] For the Risk Control GARCH a pre-defined risk level of 10% seems to be optimal. But this comes empirically with a sample volatility of about 16%.

[12] We did not make explicit assumption concerning the transaction costs. We further ignored transactions costs for replicating strategies for the capitalization weighted index in the case of changes in the constituents.

during volatile market times and vice versa, or leverage the investment. This builds on the observation of a negative correlation between the return and volatility of equity markets, i.e. very volatile periods are more often associated with negative returns. The success of the target risk strategy crucially depends on the estimate of the volatility of the risky asset for the holding period. We apply three measures, namely the sample volatility of the previous holding period, the implied volatility from option prices, and the conditional volatility derived from a GARCH(1,1) model. The capitalization weighted index serves as benchmark. We additionally place our results in the context of a minimum variance and equally-weighted strategy as alternatives. Our empirical analysis is based on the EURO STOXX 50 from 1992 to 2011. Our findings can be summarized as follows:

- The risk control strategy clearly outperforms the capitalisation weighted index. For a similar average risk level the Sharpe ratio is more than twice as high. The outperformance comes from the timing ability to leverage and deleverage depending on the market condition. It also outperforms the equally-weighted strategy.
- There is a trade-off between leverage that potentially create higher returns and the volatility loss due to leverage which leads to an optimal target risk level. This lies in the region of 15% to 20% (annualized volatility).
- The minimum variance strategy exhibits a slightly higher Sharpe ratio (compared to the Risk Control Index 15% and 20%). Both strategies also reduce the value-at-Risk. One major difference is that over time the realized volatility of the target risk control strategy stays within a narrow band whereas it heavily fluctuates for the minimum variance approach. Moreover, the average portfolio turnover at each reallocation of both strategies has about the same magnitude, while we expect the transaction costs for the minimum variance approach to be substantially higher. The reason is that individual stocks are needed to be traded in case of the minimum variance strategy, whereas for the target risk strategy it is only the index (e.g., via ETFs or futures).

- For the risk control indices more sophisticated measures to estimate future volatility levels do not lead to a performance improvement. Monthly reallocations slightly improve the performance compared to quarterly reallocations.

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